Quadratics- Questions

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

$$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

2.

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

3.

The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

4.

A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020,

(1)

(b) deduce the maximum total mass of tin that could be mined,

(1)

(c) calculate the mass of tin that will be mined in 2023.

(2)

(d) State, giving reasons, the limitation on the values of n.

(2)

5.

$$f(x) = x^2 - 8x + 19$$

(*a*) Express f(x) in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(*b*) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(*c*) Find the distance PQ, writing your answer as a simplified surd.

(3)

6.

(*a*) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \dfrac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \dfrac{1}{x} + 5$ at two distinct points,

(*b*) show that $(5 - c)^2 > 12$

(3)

(*c*) Hence find the range of possible values for c.

(4)

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7.

The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation $y = 2px^2 - 6px + 4p$, where p is a constant.

(*a*) Show that $4p^2 - 20p + 9 < 0$.

(4)

(*b*) Hence find the set of possible values of p.

(4)

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8.

The equation

$$(p - 1)x^2 + 4x + (p - 5) = 0, \text{ where } p \text{ is a constant,}$$

has no real roots.

(*a*) Show that p satisfies $p^2 - 6p + 1 > 0$.

(3)

(*b*) Hence find the set of possible values of p.

(4)

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9.

Given that $f(x) = 2x^2 + 8x + 3$,

(*a*) find the value of the discriminant of $f(x)$.

(2)

(*b*) Express $f(x)$ in the form $p(x + q)^2 + r$ where p, q and r are integers to be found.

(3)

The line $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = f(x)$.

(*c*) Calculate the value of c.

(5)

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10.

9. The equation

$$(k + 3)x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24 < 0.$$

(4)

(b) Hence find the set of possible values of k.

(3)

11.

10.

(a) Find the values of the constants a, b and c.

(3)

(b) Sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)

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12.

8. $$4x - 5 - x^2 = q - (x + p)^2,$$

where p and q are integers.

(a) Find the value of p and the value of q.

(3)

(b) Calculate the discriminant of $4x - 5 - x^2$.

(2)

(c) Sketch the curve with equation $y = 4x - 5 - x^2$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)

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13.

5. The curve C has equation $y = x(5 - x)$ and the line L has equation $2y = 5x + 4$.

(a) Use algebra to show that C and L do not intersect.

(4)

(b) Sketch C and L on the same diagram, showing the coordinates of the points at which C and L meet the axes.

(4)

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14.

7.
$$f(x) = x^2 + (k + 3)x + k,$$

where k is a real constant.

(a) Find the discriminant of f(x) in terms of k.

(2)

(b) Show that the discriminant of f(x) can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

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15.

8. The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0.$$

(3)

(b) Find the set of possible values of k.

(4)

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16.

4. (*a*) Show that $x^2 + 6x + 11$ can be written as

$$(x + p)^2 + q,$$

where p and q are integers to be found.

(2)

(*b*) Sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)

(*c*) Find the value of the discriminant of $x^2 + 6x + 11$.

(2)

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17.

10. $\mathrm{f}(x) = x^2 + 4kx + (3 + 11k)$, where k is a constant.

(*a*) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.

(3)

Given that the equation $\mathrm{f}(x) = 0$ has no real roots,

(*b*) find the set of possible values of k.

(4)

Given that $k = 1$,

(*c*) sketch the graph of $y = \mathrm{f}(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)

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18.

1. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \qquad x \neq \pm 2$$

find the values of the constants a, b, c, d and e.

(4)